## M2C3 MATH Modeling Lesson Overview

## LESSON TITLE: T-Shirt Sales

## STANDARDS ALIGNMENT:

| GRADE 3 | GRADE 4 | GRADE 5 |
| :---: | :---: | :---: |
| 3.OA: Represent and solve problems involving multiplication and division. <br> 3.OA 3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. <br> 3.OA. Multiply and divide within 100 <br> 3.OA.7: Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. <br> 3.OA Solve problems involving the four operations, and identify and explain patterns in arithmetic. <br> 3.OA. 8. Solve two-step word problems using the four operations. <br> Represent these problems using equations with a letter standing for the unknow quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. <br> 3.MD Solve problems involving measurement and | 4.OA: Use the four operations with whole numbers to solve problems. <br> 4.OA.1. Interpret a multiplication equation as a comparison, Represent verbal statements of multiplicative comparisons as multiplication equations. <br> 4.OA.2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. <br> 4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. <br> Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. <br> Gain familiarity with factors and multiples. <br> 4.OA.4. Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range $1-100$ is a multiple of a given one-digit number. Determine whether a given whole number in the range $1-100$ is prime or composite. <br> Generate and analyze patterns. <br> 4.OA.5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. | 5.OA: Write and interpret numerical expressions. <br> 5.OA 1: Us parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. <br> 5.OA 2: Write simple expressions that record calculations with numbers... <br> 5.0A. 3 Analyze patterns and relationships. <br> 5.OA.3.3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. |


| estimation...Represent and interpret data. <br> 3.MD.3: Draw a scaled picture graph or scaled bar graph to represent a data set... | 4.NF: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. <br> 4.NF.4.. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. <br> 4.NF.4.C Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem |  |
| :---: | :---: | :---: |
| MP: 1 Make sense of problem and persevere in solving them. <br> MP: 3 Construct viable arguments and critique the reasoning of others. MP: 4 Model with Mathematics | MP: 1 Make sense of problem and persevere in solving them. <br> MP: 3 Construct viable arguments and critique the reasoning of others. <br> MP: 4 Model with Mathematics | MP: 1 Make sense of problem and persevere in solving them. <br> MP: 3 Construct viable arguments and critique the reasoning of others. MP: 4 Model with Mathematics |

## CONNECTIONS (Consider while planning):

## - Previous Math Knowledge:

- Four operations (addition, subtraction, multiplication and division) using numbers between 0-1000.
- Finding patterns with whole numbers and fractions
- Adding and / or multiplying with fractions
- Making conjectures and testing results.
- Representing math ideas with pictures, symbols, and words.
- Cultural/Community/Family Connections:
- Buying items with different costs depending on the number purchased
- Shopping with family

Language Considerations: specialized terms: Buy one - Get one at half off; Buy 2 - Get one free; Better deal

## TASK:

Routine 1: Mathematizing World - Open Ended (10 minute) - Building background knowledge and connecting to funds of knowledge. [Show images of sales - Slide 2]

- What do you notice? What do these pictures make you wonder about? Brief class discussion.
- What questions do you have? What would you need to do to answer those questions?

Routine 2: Mathematizing World - Specific Math Questions (20 minute) Sensemaking and assumption building. [Show images of buy-one get-one free and buy-one get-one half off. Elicit and/or pose specific questions that can be answered using mathematics; consider using anchor chart to record "math" questions using questions stems - How much? How many? How much more/less;

Elicit questions about buying items on sale.

- What questions do you have that you could use mathematics to answer?
- What information do you need to find out how much something costs?
- Should you take advantage of the buy-one-get-one at half off, if you only need one item?

Make a plan to answer your question. Identify the information you need to collect, how you would collect that information and how you would use mathematics to answer the question.

## Routine 3: Full Modeling Task (60-90 minute) Students participate in entire modeling cycle

In this task, students will determine which T-Shirt company to buy T-Shirts from for their school.
Your school wants to buy T-shirts. Tees-R-Us has the sale "buy one, get one half off."
Sport-Mart has the sale "buy two, get one free." Which is the better deal?

Questions to think about:

- What do you know that can help you figure this out?
- What do you need to find out?
- What assumptions do you have to make?


## Relevant Information: For Teachers

To scaffold, you may provide the number of t-shirts that your school needs to buy, or use the number of students in your class. The number of $t$-shirts purchased ( $N$ ) does not make a difference once N is larger than 8 . Choosing multiples of 2 and 3 can make the solutions easier to compute, since the" buy one get one half off" works best with multiples of 2 and the "buy two, get one free" deal works best when the number of shirts is a multiple of three. Students do not need to assign a concrete price to the t-shirts to understand what "half price" means in this situation, but most will find it easier if they do so. Assigning an even dollar value will eliminate the use of fractional amounts. Teachers could choose a dollar value that results in fractional dollar values to provide opportunities for students to develop relevant fraction skills.
This task allows for multiple strategies, including different ways of representing repeated addition. Students can draw pictures, graphs, and diagrams to represent their reasoning.

## ANTICIPATED STUDENT ASSUMPTIONS:

In this task, students will have to make many decisions based on assumptions. For example, they will need to make assumptions/decisions:

## Required assumptions: You may want to make these assumptions explicit in the task if the students do not identify them through discussion.

- That both companies are selling 1 T -shirt for the same price.
- The both companies are selling the same T-shirts or T-shirts of the same quality.


## Additional Assumptions:

- They cannot answer the question unless they know the price of the T-shirts.
- They cannot answer the question unless they know how many to buy.
- The price is $\$ 1.00$ or some fixed price.
- The price is $\$ 2.00$ or some even numbered price so they do not have to work with fractional dollars.
- They will buy a certain number of T-Shirts say 24 for their class or 600 for their school and use that number to determine the answer to the question.
- If the price of the T-shirt changes then the best buy changes.


## ANTICIPATED STUDENT STRATEGIES:

- Students will give a price to one T-shirt, say $\$ 1$. They will make a table showing how the price grows. They will fill out the table and find a pattern as the number of T-shirts purchased grows. Based on their findings they will see that T's $R$ Us grows at a faster rate than Sports Mart.

| Num T's | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Cost <br> T's R Us | 1 | 1.5 | 2.5 | 3 | 4 | 4.5 | 5.5 | 6 | 7 | 7.5 | 8.5 | 9 |
| Cost <br> Sports Mart | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 | 6 | 7 | 8 | 8 |

- Students will choose number of T-Shirts they might need to buy. They may decide that for their class they would need 25 T-Shirts. They would calculate the cost of 25 for each store. For T's R Us the cost for 24 T-shirts would be $\$ 1$ for 12 shirts and $1 / 2$ of 12 for the other shirts $=\$ 12+\$ 6=\$ 18.00 \quad \$ 18 .+\$ 1=\$ 19$. for the last T-shirt. For Sports Mart the cost of $\$ 24 \mathrm{~T}$ - Shirts would be for the first $2=\$ 2$ and the $3^{\text {rd }} \mathrm{T}$-shirt is free. So, for each 3 T -shirts the cost is $\$ 2.24 / 3=8.8 x \$ 2=\$ 16$. But they want 25 T -shirts so they need to add $\$ 16+\$ 1=\$ 17$. Thus, it is less expensive to buy 24 T-Shirts at Sports Mart.
- They may notice that the numbers are easier to work with if they are multiples of 2 and 3 (or 6). They could choose larger number if they want to include all students in their grade or students in the school
Using 120 students for the T's R Us. They might notice that you divide $120 / 2=60$. 60 of the T-shirts cost 1 dollar and the other 60 cost half that. So, $\$ 60+\$ 30=\$ 90$. At

Sports Mart for 120 T-shirts, $2 / 3$ cost $\$ 1$ each but $1 / 3$ of the T-Shirts are free. $120 \times 2 / 3$ $=80$ T-shirts cost $\$ 80$ dollars and 40 T-Shirts cost $\$ 0 . \quad \$ 80+\$ 0=\$ 80$. For 120 $\mathrm{T}=$ Shirts it is less expensive to buy at Sports Mart.

- Students may notice that the more T-shirts they buy, the bigger difference they have in price. If the number of $T$-shirts purchased is small, 8 or less, sometimes it is less expensive or the same price to buy at T's R Us as Sports Mart.
- Here is an additional solution. This is an algebraic solution that is beyond the scope of grades 3-5, but you might want to use it in a PD or you might use it to guide students who would like to explore an algebraic solution.
- You can see from the table above that there are different cases depending on whether the number of T-Shirts $(\mathrm{N})$ is odd or even for T's R Us or if the number of T-Shirts ( $N$ ) has a remainder of 0 , 1 , or 2 when divided by 3 in the case of Sports Mart. The equations below outline the different cases.
- Let $\mathrm{N}=$ the number of T -shirts, C the cost of a T -Shirt, and $\mathrm{T}=$ the Total Cost of buying N T-Shirts.
- For T's $R$ Us, if $N$ is even $T=(N / 2){ }^{*} \mathrm{C}+(\mathrm{N} / 2)(\mathrm{C} / 2)=\mathrm{N} / 2(3 / 2 \mathrm{C})$. If N is odd then $\quad \mathrm{T}=((\mathrm{N}-1) / 2)(3 / 2(\mathrm{C}))+\mathrm{C}$.
- Example: If $\mathrm{N}=30$, then $30 / 2 * \$ 1+30 / 2 * \$ .50=15 * \$ 1+15 * \$ .50=\$ 15$ $+\$ 7.50=\$ 22.50$. If $\mathrm{N}=31$, then $\mathrm{N}-1=30$. The cost of $\mathrm{N}-1=\$ 22.50$. The cost of $\mathrm{N}=31=\$ 22.50+1=\$ 23.50$.
- For Sport Mart, there are different equations depending on the remainder for $N / 3$. If $N / 3$ has remainder $=0$, then $T=(2 / 3) N$ * $C$. If $N / 3$ has a remainder of 1 then $\left.T=(2 / 3)(N-1){ }^{*} \mathrm{C}\right)+\mathrm{C}$, and if $\mathrm{N} / 3$ has a remainder of 2 then $T=(2 / 3)(N-2) * C+2 C$.
- If $N=30$ then $2 / 3 N^{*} \mathrm{C}=2 / 3(30) \$ 1=\$ 20$. (20 T-shirts for 1 dollar and 10 free T-shirts.)
- If $N=31$, then $31 / 3$ has a remainder of 1 so $T=$ Cost of $30+$ Cost of $1=$ \$21.
- If $N=32$ then $32 / 3$ has a remainder of 2 , so $T=$ Cost of $30+$ Cost of $2=$ $\$ 20+\$ 2=\$ 22.00$


## MATERIALS

- T Shirt Sales_Student Task
- T Shirt Sales_Lesson Slides
- Realia - sale advertisements

